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New Method for Determining the Quality Factor and Resonance Frequency of Superconducting Micro-Resonators from Sonnet Simulations

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Abstract Lithographed superconducting microwave resonators (micro-resonators) are useful in a number of important applications, including microwave kinetic inductance detectors (Day et al., Nature 425:817, 2003), as memory elements in quantum information circuits, and as readouts of qubits and nanomechanical resonators. One of the major tasks in designing these devices is to find the resonance frequency (f_r) and quality factor (Q) for these microwave circuits using EM simulation software such as Sonnet. The traditional method iteratively runs simulations over successively smaller frequency ranges. In this way the simulated transmission S_{21} data is zoomed in on to yield a well-sampled resonance curve of a circuit. Designing microwave resonators in this manner is often time consuming since it requires many simulation runs. In this work, we show a new—and much faster—method for determining f_r and Q by adding an internal (virtual) port in the Sonnet model and examining the input impedance through the added port. Accurate f_r and Q values can be retrieved from a single simulation with a wide frequency sweep. This method works on many types of resonance circuits and dramatically reduces the simulation time.

Keywords Microwave resonator · Quality factor · Resonance frequency

1 Introduction

The commercial software Sonnet is a powerful tool that can solve planar problems by applying the Method of Moments directly to Maxwell's Equations as described on the

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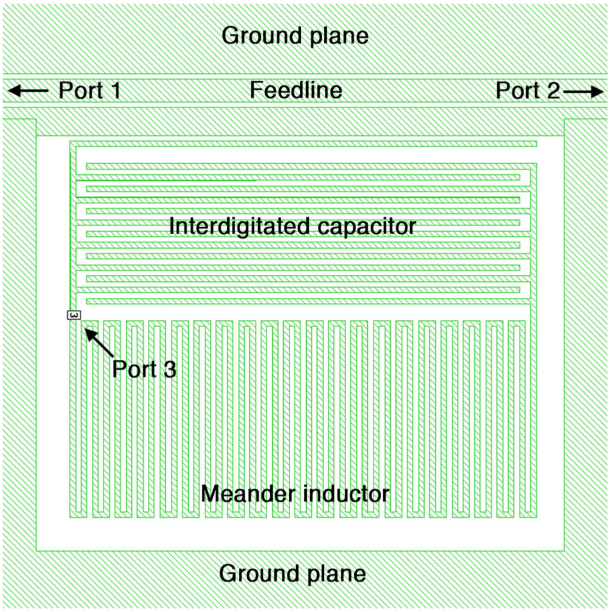

Fig. 1 An example Sonnet model with box size 1500 μm x 1000 μm and cell size of 5 μm $\times$ 5 μm. There was an air layer of 1000 μm above and a Si layer of 400 μm below the metal level. The metal sheet was lossless. The LC resonator consisted of an interdigitated capacitor with a meander inductor line. Shown here is a close up of the area with the third port. Interdigitated capacitor fingers and the meander inductor line had a width and gap of 5 μm. The fingers on the interdigitated capacitor had a length of 400 μm while the length of one winding in the meander line was 175 μm with a total of 42 windings. Also shown is a virtual third port added to the model in between the interdigitated capacitor and the meander inductor line (Color figure online)

company's website [2]. All simulations were done using Sonnet version 14.52, the most current at the time. Simulating microwave resonators using the popular EM simulation software Sonnet is a time consuming process, particularly when only a rough idea of the resonance frequency (f_r) or quality factor (Q) is known beforehand. The traditional 2-port method iteratively runs multiple simulations over successively smaller frequency ranges and has many disadvantages. This paper presents a new 3-port method and the result is compared with the traditional 2-port method on an example LC microwave resonator model shown in Fig. 1, which is a lumped element kinetic inductance detector (LEKID). Determining the resonance frequency and quality factor of a microwave resonator can now be accomplished in a single sweep using the new method described in this paper.

2 The 2-Port Method

Design of the LEKIDs in this work were based on the previous work of Doyle et al. [3]. The standard 2-port method for the "feedline-resonator" model of an LEKID design is illustrated in Fig. 2. The conventional procedure to obtain f_r and Q is as follows:

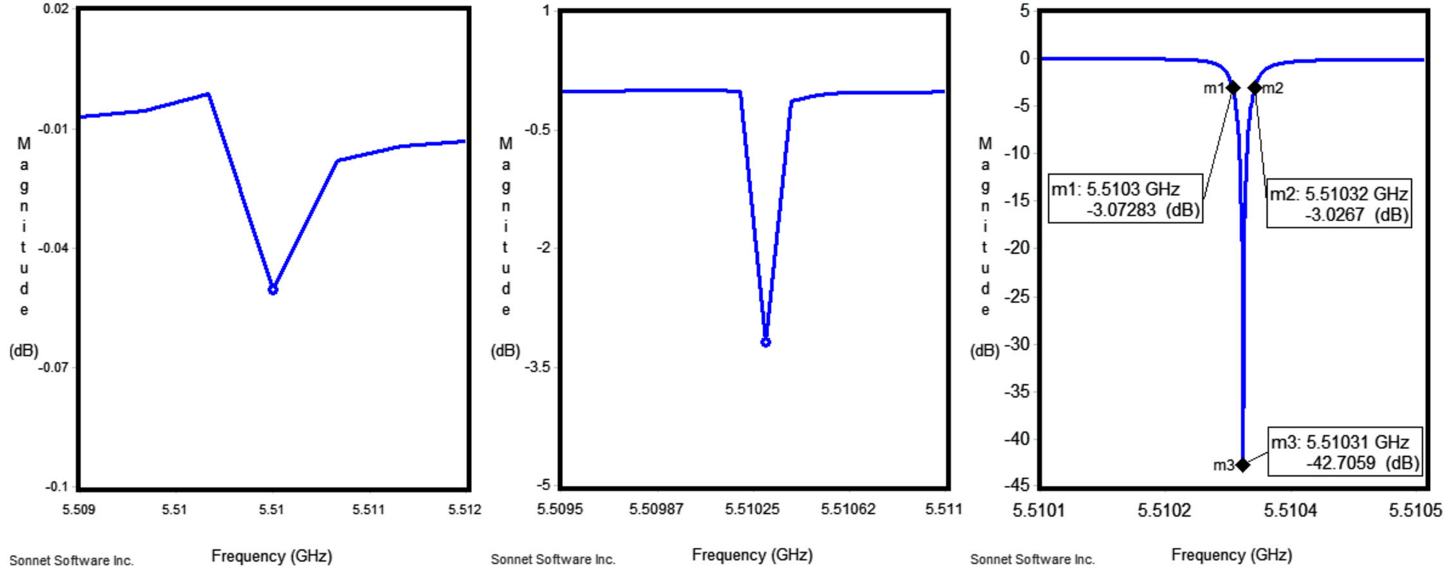

Fig. 2 (*left*) S_{21} from the coarse sweep between 5.509 GHz and 5.512 GHz. (*middle*) Refined sweep over small frequency range. (*right*) Closeup showing the refined sweep over the same resonance. A quality factor of Q = 240,000 is obtained from the 3-dB bandwidth (Color figure online)

Step 1: Run a sweep over a wide frequency range in which the resonance is expected (Fig. 2a had an initial frequency range of 5–6 GHz). Find the signature of the resonance. Often after zooming in, one finds that it is still under-sampled (Fig. 2b).

Step 2: Run successively refined sweeps over narrower frequency ranges until the resonance is sufficiently pronounced. Q can be retrieved by finding the 3-dB bandwidth of the Lorentzian curve (Fig. 2c) or by resonator fitting method. If the metal sheet is originally assigned with no loss, this derived Q is essentially the coupling Q_c of the resonator design.

This procedure involves many memory-intensive and time-consuming simulation runs. Particularly if the grid size in Sonnet is set to 1 μm as is often the case for actual designs to ensure the feed line has an impedance of 50 Ω. One design similar to this could take as much as 30 min to an hour depending on the computer. The initial pass of the sweep may even miss the resonance frequency, especially for high Q devices.

3 The 3-Port Method

In the new method, a third internal port is inserted into an LEKID at the spot where the interdigitated capacitor joins the meander inductor, as shown in Fig. 3a. With the other two ports (port-1 and port-2) terminated by matching loads, the resonator can now be treated as an RLC resonance circuit when looking into the third port. A standard expression for the impedance of a series RLC circuit, according to Pozar [4], is

$$Z_{in,3} = j\omega L + \frac{1}{j\omega C} + R \tag{1}$$

where L and C are the inductance and capacitance of the resonator. In our case, R is an effective series resistor arising from the dissipation of the loads at port-1 and port-2. The value of R depends on the coupling strength between the resonator and the feedline. Near the series resonance, the impedance takes the form

$$Z_{in,3} = R\left[1 + 2jQ(\frac{f - f_r}{f_r})\right] \tag{2}$$

Equation 2 is the general expression for impedance near the resonance frequency according to Pozar [4] for a series RLC circuit. At the resonance frequency $Z_{in,3} = R$ and can be read from the data in Fig. 3 where Im[Zin,3] crosses zero. This expression is more general because it applies to either a distributed or a lumped element circuit near one or more resonance frequencies. The derivation proving that this applies to both lumped element and more distributed element circuits is beyond the scope of this paper. However, we verified with Sonnet that this method can find the resonance frequency and give an accurate approximation of the quality factor for both a CPW and LEKID.

Equation 2 suggests another way of finding f_r and Q. The resonance frequency f_r occurs where Im[Zin,3] crosses zero (Fig. 3c). When the imaginary part vanishes at f_r, the real part of $Z_{in,3}$ directly gives the value of R. We note that Eq. 2 is linear

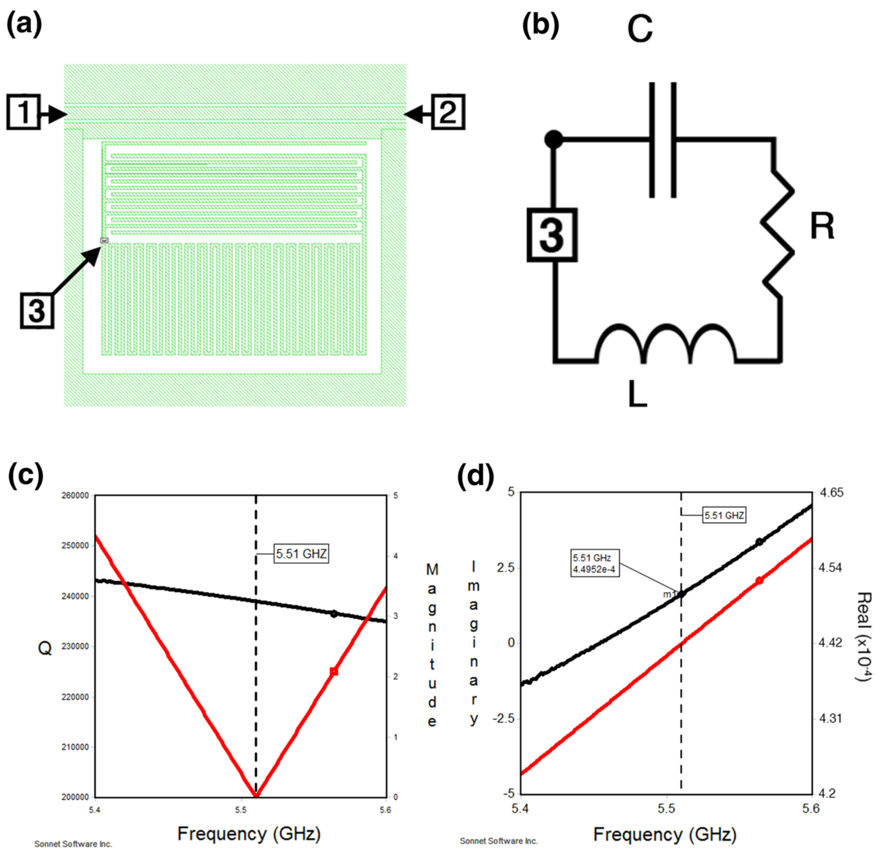

Fig. 3 **a** Image of the LEKID used in the Sonnet simulations showing the third port. **b** Circuit diagram with the added 3-port. **c** Magnitude of Zin,3 (*black*) and Q calculated from a custom defined curve equation "QZin3" (*grey* or *red* online in color) as a function of frequency. **d** Real (*black*) and imaginary (*red*) parts of $Z_{in,3}$ as a function of frequency. The value of Q = 239, 000 was found using the custom defined curve shown in **(c)** (Color figure online)

in f. The value of Q is related to the local (linear) slope k of the imaginary part $Z_{in,3}$(f) by

$$Q = \frac{kf_r}{2R}. \quad (3)$$

The following steps were carried out to complete the 3-port method for the LEKID shown in Fig. 1

Step 1: Cut the inductor using the command "divide polygon" and insert an internal port (port-3) as shown in Fig. 3a.

Step 2: Simulate this 3-port model over a wide frequency range in which the resonance is expected. We chose between 4.5 and 6.5 GHz for this example. By starting at a very low frequency, one can effectively find the lowest frequency without having to worry about missing anything.

Step 3: Plot the real and imaginary part of $Z_{in,3}$(f) as shown in Fig. 3d. From the zero-crossing of Im[$Z_{in,3}$(f)] (red), we get fr = 5.51 GHz. From the Re[$Z_{in,3}$(fr] curve (black), we obtain R = Re[$Z_{in,3}$(fr)] = 0.00045. The slope k of Im[Zin,3(f)] can be estimated from two adjacent points on the Im[Zin,3(f)] curve, or more accurately by a linear fit. Finally, Q is obtained from Eq. 3.

Using the "equation curve" feature of Sonnet may make step 3 even more efficient. A Sonnet curve equation "QZin3" is defined to calculate Q, which implements Eq. 3. The plots of both magnitude of $Z_{in,3}$ and this "Q-curve" are shown in Fig. 3c. In Fig. 3c, f_r is clearly indicated by the dip (minimum) in the mag($Z_{in,3}$) curve (equivalent to finding the zero crossing in the imaginary part of $Z_{in,3}$ in Fig. 3d) and the value of Q can be read off of the "Q-curve" at the frequency (f_r). The results are shown in Fig. 3c which gives a quality factor of 239,000. The quality factors for the 2-port method (Q_2 port = 240, 000) and the 3-port method (Q_3 port = 239, 000) agree well.

The advantage of the new method is that it requires only one sweep over a wide frequency range which drastically decreases the simulation time required for designing a microwave resonator. An equation curve can be inserted in Sonnet so that the Q-value can be calculated automatically. The code for performing this has been omitted and can be found online on Dr. Wisbey's website [5].

4 Discussion

The new 3-port method was also used to determine the resonance frequency for a coplanar waveguide microwave resonator and it was found that it works equally well. In general, this method applies to lumped element resonator circuits and also distributed circuits and other resonator types, such as the popular quarter-wave CPW resonator. The 3-port method should be valid in general for circuits that behave like a notch type filter. If the frequency range of the sweep is wide enough, one may also find higher order resonances (harmonics) of the circuit and the associated Qs with each resonance, all in one sweep. We also found that the 2-port and 3-port method predicted the resonance frequency to within 0.5 % of each other.

In the LEKID model, the third port was inserted between the inductor and the capacitor as shown in Fig. 1. In fact, the choice of the third port is not unique. In general, it should be inserted at a current maximum in order to represent the impedance near resonance of a series RLC circuit. For a quarter-wave CPW resonator, the third port can be inserted at the shorting end where the center line joins the ground plane. A different location of the third port may change the detailed shape of the input impedance curve, but it will not change the results of f_r and Q.

If the internal port is shorted out (by setting the Z_0 of the third virtual port to R = $10^{-8}\Omega$) and only the S_{21} transmission is looked at, one would expect that the 3-port model reduces to the 2-port model. Indeed, we find that the S_{21} curve from the 3-port simulation, with the third port shorted out, overlaps identically with the S_{21} curve from the 2-port simulation. This further verifies that the old 2-port method and the new 3-port method lead to the same results.

5 Conclusions

This new 3-port simulation method significantly reduced the time necessary to design and characterize microwave resonators. Resonance frequencies and quality factors found with the 3-port method agreed with the results from the 2-port method. By defining an “equation curve” in Sonnet, one can simply read off the resonance frequency and quality factor of a microwave resonator after only a single simulation.

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